

# A NEW WAY OF PLANNING SETI SEARCHES BY THE STATISTICAL DRAKE EQUATION

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# ABSTRACT

- ▶ We provide the statistical generalization of the Drake equation.
- ▶ From a simple product of seven positive numbers, the Drake equation is now turned into the product of seven positive random variables.
- ▶ We call this “the Statistical Drake Equation”.
- ▶ A key advantage is to associate an **ERROR BAR** to each variable in the classical Drake equation.

# *The Classical Drake Equation /1*

- ▶ In 1961 Frank Drake introduced his famous “Drake equation” described at the web site [http://en.wikipedia.org/wiki/Drake\\_equation](http://en.wikipedia.org/wiki/Drake_equation). It yields the number  $N$  of communicating civilizations in the Galaxy:

$$N = N_s \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot f_L$$

- ▶ Frank Donald Drake (b. 1930)



# *The Classical Drake Equation /2*

- ▶ All Attendees in this meeting do know the meaning of the seven factors in the Drake equation: so, they will not be described here.
- ▶ On the contrary, we'd like to stress that the seven factors are just **POSITIVE NUMBERS**. And that the Drake equation simply is the **PRODUCT** of these seven positive numbers.
- ▶ It is claimed here that Drake's approach is too "simple-minded", since it does NOT yield the **ERROR BAR** associated to each factor!

# *The STATISTICAL Drake Equation /1*

- ▶ If we want to associate an **ERROR BAR** to each factor of the Drake equation then...
- ▶ ... we must regard each factor in the Drake equation as a **RANDOM VARIABLE**.
- ▶ Then the number  $N$  of communicating civilizations also becomes a random variable.
- ▶ This we call the **STATISTICAL DRAKE EQUATION** and study for the first time here.

# *The STATISTICAL Drake Equation /2*

- ▶ Denoting each random variable by capitals, the **STATISTICAL DRAKE EQUATION** reads

$$N = \prod_{i=1}^7 D_i$$

- ▶ Where the  $D_{sub\ i}$  ("D from Drake") are the 7 random variables, and  $N$  is a random variable too ("to be determined").

# *The STATISTICAL Drake Equation /3*

- ▶ The next question is: what is the statistical distribution of each  $D_{sub\ i}$ ?
- ▶ To simplify things, we'll just suppose that each  $D_{sub\ i}$  is **UNIFORMLY DISTRIBUTED**.
- ▶ Thus, the **MEAN VALUE** of each  $D_{sub\ i}$  has same numeric value as the corresponding pure number in the Drake equation !
- ▶ Each  $D_{sub\ i}$  also has an **ERROR BAR** around the mean value that is equal to the standard deviation of the UNIFORM distribution !

# *The STATISTICAL Drake Equation /4*

- ▶ For instance: nobody knows what the number of stars in the Galaxy is exactly! 😊
- ▶ But in our Statistical Drake Equation, we avoid this rough approximation by replacing the unknown number of stars by a UNIFORM distribution centered around the most probable number. Also, it spreads out to the value of the standard deviation of the uniform distribution.
- ▶ It can be proven that the latter must be multiplied by a factor of  $\sqrt{3} = 1.7$ .

# *The STATISTICAL Drake Equation /5*

- ▶ This author has completely solved the problem of finding the statistical distribution of  $N$  given the 7 uniformly distributed random variables  $D_{sub\ i}$ . It does not have a name and is rather complicated.
- ▶ The mathematical proof is too long to be described here in detail (sorry). It will be published soon.
- ▶ Let us further try to GENERALIZE the Statistical Drake Equation to the case where the random variables are OTHER THAN SEVEN. Then...

# *Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /1*

- ▶ Consider the statistical equation

$$N = \prod_{i=1}^{\text{any number}} D_i$$

- ▶ This is the generalization of our Statistical Drake Equation to the product of ANY finite NUMBER of positive random variables.
- ▶ Is it possible to determine the statistics of  $N$ ?
- ▶ Rather surprisingly, the answer is "yes" !

# *Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /2*

- ▶ First, you obviously take the natural log of both sides to change the finite product into a finite sum

$$\ln(N) = \sum_{i=1}^{\text{any number}} \ln(D_i)$$

- ▶ Second, to this finite sum one can apply the **CENTRAL LIMIT THEOREM OF STATISTICS**. It states that, in the limit for an infinite sum, the distribution of the left-hand-side is **NORMAL**.
- ▶ This is true **WHATEVER** the distributions of the random variables in the sum **MAY BE** *Just Great!*

# *Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /3*

- ▶ So, the random variable on the left is **NORMAL**, i.e.

$$\ln(N)$$

- ▶ Thus, the random variable  $N$  under the log must be **LOG-NORMAL** and its distribution is determined!
- ▶ One must, however, determine the mean value and variance of this log-normal distribution in terms of the mean values and variances of the factor random variables. This is **DIFFICULT**. But it can be done, for example, by a suitable numeric code that this author wrote in MathCad language.

# Conclusion #1

## *The number of Signaling Civilizations is Log-Normally distributed*

- ▶ Our Statistical Drake Equation, now Generalized to any number of factors, embodies as a special case the Statistical Drake Equation with just 7 factors.
- ▶ The conclusion is that the random variable  $N$  (the number of communicating ET Civilizations in the Galaxy) is **LOG-NORMALLY distributed**.
- ▶ The classical “old pure-number Drake value” of  $N$  is now replaced by the **MEAN VALUE** of such a log-normal distribution.
- ▶ But we now also have an **ERROR BAR** around it !

# Example #1

## UNIFORM INPUT DISTRIBUTIONS

$$N_s := 350 \cdot 10^9$$

$$\mu_{N_s} := N_s$$

$$\sigma_{N_s} := 1 \cdot 10^9$$

$$f_p := \frac{50}{100}$$

$$\mu_{f_p} := f_p$$

$$\sigma_{f_p} := \frac{10}{100}$$

$$n_e := 1$$

$$\mu_{n_e} := n_e$$

$$\sigma_{n_e} := \frac{1}{\sqrt{3}}$$

$$f_l := \frac{50}{100}$$

$$\mu_{f_l} := f_l$$

$$\sigma_{f_l} := \frac{10}{100}$$

$$f_i := \frac{20}{100}$$

$$\mu_{f_i} := f_i$$

$$\sigma_{f_i} := \frac{10}{100}$$

$$f_c := \frac{20}{100}$$

$$\mu_{f_c} := f_c$$

$$\sigma_{f_c} := \frac{10}{100}$$

$$f_L := \frac{10000}{10^{10}}$$

$$\mu_{f_L} := f_L$$

$$\sigma_{f_L} := \frac{1000}{10^{10}}$$

$$N := N_s \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot f_L$$

$$f_L\_factor := N_s \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c$$

$$\mu_{f_L} = 0.000001$$

$$N = 3500$$

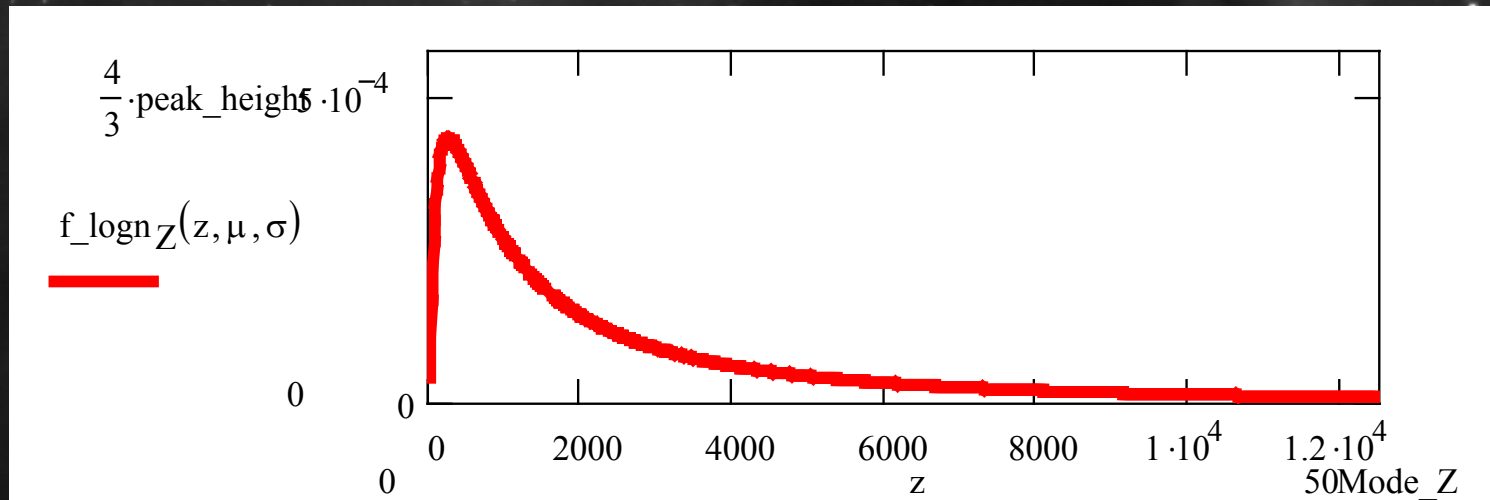
$$f_L\_factor = 3500000000$$

$$\sigma_{f_L} = 0.0000001$$

# Example #1

## The number of Signaling Civilizations is Log-Normally distributed

- ▶ For the above uniform inputs, the log-normal curve



- ▶ Yields the number of Signaling Civilizations (mean value) as 4590, while classic Drake eq. yields 3500.
- ▶ With a standard deviation of  $\pm 11,195$  Civilizations.

## Conclusion #2

# *The DISTANCE between ET Civilizations now follows a probability distribution*

- ▶ In all Astrobiology textbooks (e.g. Bennett & Shostak 2007 ed., page 404) the average **DISTANCE** between Signaling Civilizations in the Galaxy is given. This is inversely proportional to the cubic root of the number  $N$  obtained by the classical Drake equation.
- ▶ Now this average distance is a new random variable. This author found the relevant probability distribution. But this is too complicated to be described here.
- ▶ The **MEAN VALUE** of such a new distribution is the old average **DISTANCE**, but we now also have an **ERROR BAR** around it !

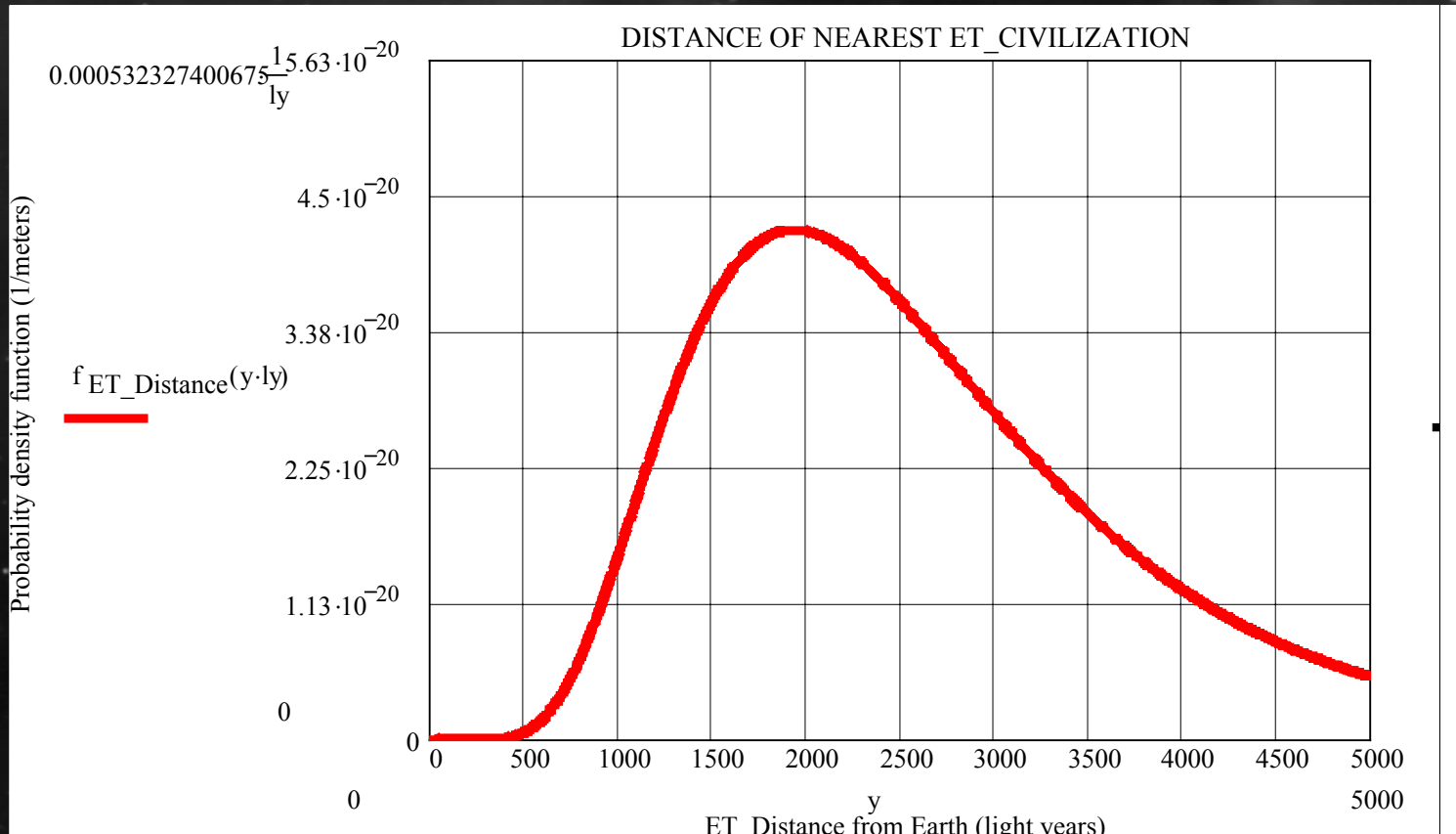
## Example #2

*The DISTANCE between ET Civilizations now follows a probability distribution*

- ▶ In the previous example, the average DISTANCE given by the classic Drake equation is 4093 light years.
- ▶ In our statistical approach, the **MEAN VALUE** of the distance turns out to be 2670 light years...
- ▶ ...with an **ERROR BAR** of  $\pm 1309$  light years.
- ▶ Thus the MINIMAL STATISTICAL ET DISTANCE is JUST 1361 light years. This is SETI-ENCOURAGING!

# Example #2

*The DISTANCE between ET Civilizations now follows a probability distribution*



# CONCLUSION

- ▶ Surprisingly, **STATISTICS** and the Drake equation **MATCH PERFECTLY** if one just takes the logs of the Drake equation!
- ▶ By doing so, one also invokes the **Central Limit Theorem (CLT) of Statistics**, enabling ANY number of factors to be considered.
- ▶ This is our **DATA ENRICHMENT PRINCIPLE** that is the key to further future Astrobiology investigations by virtue of our extension.



***Thanks very much !***